Radial Limits of Nonparametric Prescribed Mean Curvature surfaces

Mozhgan (Nora) Entekhabi
Department of Mathematics, Statistics & Physics
Wichita State University
Wichita, Kansas, 67260-0033

Abstract

Consider a bounded solution f of the prescribed mean curvature equation over a bounded domain $\Omega \subset \mathbb{R}^2$ which has a corner at $(0,0)$ of size 2α and assume the mean curvature of the graph of f is bounded. If the corner is nonconvex/reentrant (i.e. $\alpha \in \left(\frac{\pi}{2}, \pi\right))$), then the radial limits

$$Rf(\theta) \stackrel{\text{def}}{=} \lim_{r \downarrow 0} f(r\cos\theta, r\sin\theta)$$

exist for all interior directions (e.g. $\theta \in (-\alpha, \alpha)$ if $\theta = \pm\alpha$ are tangent rays to $\partial\Omega$ at $(0,0)$), no matter how wild is the trace of f on $\partial\Omega$. If the corner is convex/protruding (i.e. $\alpha \in \left(0, \frac{\pi}{2}\right]$) and some extra conditions are satisfied, then the radial limits at $(0,0)$ from interior directions continue to exist. This generalizes, for example, known results about radial limits of capillary surfaces.