The periodic Dirac operator is absolutely continuous

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We fix a lattice Γ in $\mathbf{R}^d$, $d \geq 2$. The elementary cell of the lattice is denoted by Ω; $D_j = -i\partial/\partial x_j$. Let $\alpha_1, \cdots, \alpha_{d+1}$ be Dirac matrices, i. e. Hermitian $(M \times M)$-matrices, $M = 2^{d-1}$, which satisfy the following relations

$$\alpha_j \alpha_l + \alpha_l \alpha_j = 2\delta_{jl}\mathbf{1}, \quad j, l = 1, \ldots, d+1,$$

where $\mathbf{1}$ is the unit matrix. Let $V(\mathbf{x})$, $V_0(\mathbf{x})$ be scalar-valued potentials and let $\mathbf{A}(\mathbf{x}) = \{A_1(\mathbf{x}), \ldots, A_d(\mathbf{x})\}$ be a vector-valued (magnetic) potential. *All the potentials are real and* Γ*-periodic.* The function V is the electric potential; usually $V_0 = m =$ const, where m is the mass of a particle, but it is more convenient to consider the general case. One can always assume that the magnetic potential satisfies the gauge conditions

$$\operatorname{div} \mathbf{A} = 0, \quad \int\limits_\Omega \mathbf{A}(\mathbf{x})\, d\mathbf{x} = 0; \tag{1}$$

the first equality is understood in the sense of distributions. We assume that

$$\mathbf{A} \in L_r(\Omega), \quad V_0 \in L_r(\Omega), \quad V \in L_r(\Omega), \quad r > 2, \quad d = 2, \tag{2}$$

$$\mathbf{A} \in H^s(\Omega), \quad s > 3d/2 - 1, \quad d \geq 3, \tag{3}$$

$$V_0, \quad V \in C(\mathbf{R}^d), \quad d \geq 3. \tag{4}$$

Under these assumptions **the Dirac operator**

$$\mathcal{D}(\mathbf{A}, V_0, V) := \sum_{j=1}^{d} (D_j - A_j(\mathbf{x}))\alpha_j + V_0(\mathbf{x})\alpha_{d+1} + V(\mathbf{x})\mathbf{1}, \quad d \geq 2, \tag{5}$$

is selfadjoint in $L_2(\mathbf{R}^d)$ on the domain $H^1(\mathbf{R}^d)$.

In the case $\mathbf{A} = 0$, $V_0 = 0$, $V \in C(\mathbf{R}^d)$, $d \geq 3$, the absolute continuity of $\mathcal{D}$ has been proved by L. Danilov [1]. The result for the Dirac operator with magnetic potential has been obtained by the authors [2]. The latter case is essentially more difficult.

Our main result is the following theorem.

Theorem 1. *Under the assumptions* (1), (2) *for* $d = 2$ *or* (1), (3), (4) *for* $d \geq 3$, *the Dirac operator* (5) *is absolutely continuous.*

The condition (2) for $d = 2$ seems to be optimal in L_p-scale, while the conditions (3) and (4) for $d \geq 3$ are far from being optimal. The latter is related to the technique of the proof. The proof is heavily based on the results on the absolute continuity of the periodic magnetic Schrödinger operator, obtained by the authors [3, 4] for $d = 2$ and by A. Sobolev [5] for $d \geq 3$.

References

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