Ground State Energy of the Low Density Fermi Gas

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Recent developments in the physics of low density trapped gases make it worthwhile to verify old, well known results that, while plausible, were based on perturbation theory and assumptions about pseudopotentials. We use and extend recently developed techniques to give a rigorous derivation of the asymptotic formula for the ground state energy of a dilute gas of N fermions interacting with a short-range, positive potential of scattering length a. For spin 1/2 fermions, this is $E \sim E^0 + (\hbar^2/2m)2\pi N\rho a$, where E^0 is the energy of the non-interacting system and ρ is the density. This theorem is harder to prove than the corresponding result for bosons (namely $E \sim (\hbar^2/2m)4\pi N\rho a$) because the interaction energy is only the second term in the energy for fermions instead of the leading term, as it is for bosons. This is joint work with R. Seiringer and J.P. Solovej; arXiv math-ph/0412080.