Geometric optics and the wave equation on manifolds with corners

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An attractive picture of how singularities of solutions of the wave equation propagate is that they obey the laws of 'geometric optics': they follow straight lines (or geodesics) until they encounter obstacles or interfaces, where they reflect/refract according to Snell's law. More precisely, both the singularities of solutions and the geometric optics rays lie in 'phase space', which is the cotangent bundle for manifolds without boundary; this is the origin of the name 'microlocal analysis'. A modern version of this theory was initiated with Keller, but has proved surprisingly difficult to substantiate mathematically. The true behavior of singularities of waves when they hit smooth boundaries was only proved in some generality in the 1970's by Melrose and Sjöstrand, and Taylor; Lebeau settled the case of manifolds with corners in the mid 1990's, but only when all data is analytic.

I shall discuss my recent general theorem regarding propagation of singularities on arbitrary (smooth) manifolds with corners. Both the result and its proof are related to the quantum scattering of N particles, with geometric optics replaced by a slightly modified version of classical mechanics to account for bound states of particles, and $N-2$ corresponding to the highest codimension of a corner. I shall try to give an overview of how such theorems are proved, focusing on the so-called positive commutator method.