Irregular Sampling of Band-limited Signals

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Let $f(t) \in L_2(R)$ and $\hat{f}(\lambda)$ be its Fourier transform. If *supp* $\hat{f} \in [-T, T]$, f is a band-limited signal with the band-width T.

The fundamental of digital signal processing is the Shannon sampling formula

$$f(t) = \sum_{n=-\infty}^{\infty} f\left(\frac{n\pi}{T}\right) \frac{\sin(Tt - n\pi)}{Tt - n\pi},$$

that allows to recover a band-limited signal with band-width T from its samples at equidistant points spaced $\frac{\pi}{T}$ apart.

Now suppose that our receiver gets a signal from a unknown source. So no information about the band-width of the signal is known. Moreover, the sampling points are irregularly distributed. What conditions should we put on a set of sampling points so that we can recover any band-limited signal f?

A solution of this problem is based on the inverse spectral problem and Kramer's theorem. The connection between the two is that given a sequence of points, satisfying certain distribution law, we can construct a singular Sturm-Liouville problem such that these points are precisely its eigenvalues. For this particular Sturm-Liouville problem, Kramer's theorem then allows us to use the spectral theorem as a sampling formula.