Quasi-linear and Hessian equations with non-linear source terms

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We establish necessary and sufficient conditions for the existence of admissible solutions to quasilinear and Hessian equations with nonlinear source terms, including the following two model problems:

$$-\Delta_p u = u^q + \mu, \qquad F_k[-u] = u^q + \mu, \qquad u \geq 0,$$

on $\mathbb{R}^n$, or on a bounded domain $\Omega \subset \mathbb{R}^n$. Here Δ_p is the p-Laplacian defined by $\Delta_p u = \operatorname{div}(\nabla u |\nabla u|^{p-2})$, and $F_k[u]$ is the k-Hessian defined as the sum of $k \times k$ principal minors of the Hessian matrix $D^2 u$ ($k = 1, 2, \ldots, n$); μ is an arbitrary non-negative measurable function (or measure) on Ω. These results lead to a complete characterization of removable singularities, universal estimates and Liouville-type theorems for solutions understood in the p-superharmonic or k-convex sense.

Our approach is based on a systematic use of Wolff's potentials and nonlinear trace inequalities, along with recent advances in potential theory and PDE due to Kilpeläinen and Malý, Trudinger and Wang, and Labutin. This enables us to treat nonlocal operators, singular solutions, and distributed singularities, and develop the theory simultaneously for quasilinear equations and equations of Monge-Ampère type. This talk is based on joint work with Igor E. Verbitsky.