Oscillations of Eigenfunctions of Self-Adjoint Fourth-Order Two Point Boundary Problems with Negative Eigenvalues

Jamel Ben Amara
Faculte des Sciences de Bizerte, Tunisia

In this work, we investigate the oscillatory properties of the natural modes of oscillation for a vibrating beam which is fixed at one end. From elementary beam theory, these modes are the eigenfunctions determined by the differential equation

$$l(y) = (py")" - (qy')' = \lambda ry, \qquad ' := \frac{d}{dx}. \tag{1}$$

subject to the boundary conditions

$$y(0) = y'(0) = 0 \quad \text{(clamped end)} \tag{2}$$

$$y'(1)cos\gamma + (py")(1)sin\gamma = 0 \tag{3}$$

$$y(1)cos\delta - Ty(1)sin\delta = 0, \tag{4}$$

where

$$Ty = (ry")' - qy' \tag{5}$$

and $0 \leq \gamma, \delta \leq \pi$. The coefficients $p > 0$, q and $r > 0$ are assumed to be sufficiently smooth to ensure the existence of the eigenfunctions. Banks and Kurowski [1], [2], using an extension of the Prufer transformation, have studied this problem for $0 \leq \gamma, \delta \leq \frac{\pi}{2}$ and $q > 0$. They showed that all the eigenvalues are positive and simple and the corresponding eigenfunctions satisfy the well known Sturm-oscillation criterion [3]. Here, we are mainly interested in the case where $\frac{\pi}{2} \leq \gamma, \delta \leq \pi$. We show that for $q > 0$, the above problem can have at most two negative eigenvalues where the corresponding eigenfunctions have an arbitrary number of zeros in $(0, 1)$. It is known from many examples that if q changes sign in $(0, 1)$, then multiple eigenvalues can occur. However, we show that the principal eigenvalue of the problems defined by (1),(2) and one of the conditions $y'(1) = Ty(1) = 0$, $py''(1) = Ty(1) = 0$, $y(1) = py''(1) = 0$ is simple and the corresponding eigenfuction has a constant sign in $(0, 1)$.

REFERENCES

[1] D.O.Banks, G.J.Kurowski, *A prufer transformation for the equation of the vibrating beam*, Trans. Amer. Math. Soc. (1974), vol. 199, pp.203-222.

[2] D.O.Banks, G.J.Kurowski, *A prufer transformation for the equation of the vibrating beam with axial forces*, J. diffent.Eqution. (1977), vol. 24,, pp.75-98.

[3]C. Sturm, *Sur une classe d'equations á derivees partielles*, J. Math. Pures Appl. (1836), vol. 1, pp. 373-444.