Integrable Magnetic Vortex Dynamics and Complex Burgers' Equation

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In this work we consider a modification of the Heisenberg model proposed by Volovik for restoration of the correct linear momentum density of the ferromagnets. In analogy with superfluid motion in He3 he introduced the normal velocity of the fermionic liquid as an additional hydrodynamic variable, describing the background fermionic vacuum. According to the Mermin-Ho relation the vorticity of the fluid has to be proportional to the magnetic topological current. We show that the stereographic projection of the spin phase space admits reduction in terms of time dependent holomorphic functions subject to the complexified linear Schrodinger equation with harmonic potential. For constant and linear potentials, solution with N- zeroes describes magnetic vortices moving in the plane. It is shown that vortex dynamics can be described by multiparticle system corresponding to the reduced complex Calogero-Moser system. Expanding solutions in terms of complex Hermite polynomials we construct N- vortex configurations with arbitrary N. By using complex version of the Cole-Hopf transformation we show that model leads to direct linearization of 2+1 dimensional Burgers' equation with analytic space dependence. Applying above results, exact solutions of complex Burgers' equation of exponential type (solitons) and the polynomial type (vortices) are found. (This results are obtained in collaboration with Oktay Pashaev)