Symmetry Analysis of a System of Anisotropic Plane Plasticity

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The classical system of plane plasticity with a general yield criterion $f(\sigma, \tau) = 0$ has the following form [1]

$$\begin{aligned} &\sigma_{x_1} - 2\tau\,(\theta_{x_1}\cos 2\theta + \theta_{x_2}\sin 2\theta) = \tau_{x_1}\sin 2\theta - \tau_{x_2}\cos 2\theta, \\ &\sigma_{x_2} - 2\tau\,(\theta_{x_1}\sin 2\theta - \theta_{x_2}\cos 2\theta) = -\tau_{x_1}\cos 2\theta - \tau_{x_2}\sin 2\theta, \end{aligned} \tag{1}$$

where σ is a hydrostatic pressure, τ is a maximal shear stress, $\theta + \pi/4$ is the angle between the first principal direction of a stress tensor and the ox_1-axis, indices mean the derivation with respect to corresponding variables.

This system describes a plastic plane deformation of anisotropic materials, in particular, is applying in the static of soil medias. In the case when $\tau = \tau(\theta)$ the system (1) has the form

$$\begin{aligned} &\sigma_{x_1} - \theta_{x_1}\,[\tau\sin 2\theta]'_\theta + \theta_{x_2}\,[\tau\cos 2\theta]'_\theta = 0, \\ &\sigma_{x_2} + \theta_{x_1}\,[\tau\cos 2\theta]'_\theta + \theta_{x_2}\,[\tau\ \sin 2\theta]'_\theta = 0, \end{aligned} \tag{2}$$

and is a hyperbolic one for any form of the function τ, therefore this form of dependence is used widely in the theory of plasticity [2].

In the report we'll present a group classification of the system (2) using well-known methods [3]. There were found some specifications of τ, for which we have the extension of a basic group of symmetries. For Lie algebras, corresponding to this specifications the optimal systems of subalgebras were constructed and some invariant solutions were obtained.

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References

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[3] L.V. Ovsyannikov, *Group Analisis of Diferential Equations*, New York, Academic Press, 1982.