Direct And Inverse Spectral Problems For Differential Operators With Singular Boundary Conditions

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We consider direct and inverse spectral problems for a class of singular differential equations of the form

$$-\frac{d}{dt}\Big(p_2(t)\frac{dz}{dt}\Big) + p_1(t)z(t) = \lambda p_0(t)z(t), \quad t \in (t_0, t_1).$$

Here λ is the spectral parameter, and the real-valued functions $p_k(t)$ have zeros or/and singularities at the end points of the interval (t_0, t_1). More precisely,

$$p_k(t) = (t - t_0)^{s_{k0}}(t_1 - t)^{s_{k1}} p_{k0}(t),$$

where s_{km} are real numbers, $p_{10}(t) \in C[t_0, t_1]$, $p_{k0}(t) \in C^2[t_0, t_1]$, $p_{k0}(t) > 0$ for $k = 0, 2,\ t \in [t_0, t_1]$. We assume that $s_{2m} < s_{0m} + 2$, $s_{2m} \le s_{1m} + 2$, $m = 0, 1$, i.e. we consider the case of the so-called regular singularities.

In the first part of our lecture we provide a general method for defining two-point singular boundary conditions in the above-mentioned general case. Using this definition we derive asymptotic estimates for the eigenvalues of a corresponding class of boundary value problems and various properties of the set of eigenfunctions.

Subsequently we study the corresponding inverse spectral problems for the case of *separated* singular boundary conditions and also for a class of *non-separated* singular boundary conditions. For each class we prove a uniqueness theorem and give a procedure for constructing the solution of the inverse problem.

This is a report on joint work with V. Yurko, Saratov, Russia.