Exact Solution of the Dirichlet and Neumann problems for nonlinear Poisson equation on the plane.

Igor Tralle
University of Rzeszów, Poland

The method for searching of the solution to Dirichlet problem for nonlinear Poisson equation (NPE) on the plane is proposed. We prove that the Dirichlet problem for NPE on the plane can be solved for almost arbitrary simply connected domain which can be mapped conformally into the unit circle. If the function which maps this domain conformally into the circle is known, the solution can be expressed in an explicit form derived in the paper. Our approach is based on the observation that there exists *Bäcklund transformation* which interrelates the solutions of NPE with the solutions of Laplace equation. Thus, in order to solve the Dirichlet problem for NPE, one should solve an appropriate auxiliary boundary value problem for the Laplace equation. This boundary value problem can be easily formulated for the cirlce; as it turns out, it is *Cauchy problem* and in order to be consistent, the boundary condition must be represented by the π-periodic function. This result is generalized to the solution of the Dirichlet problem for NPE in almost arbitrary simply connected domain on the plane. The solution of Neumann problem for NPE is also considered. Because of the constraints imposed by *Green's Theorem* on the outward normal derivative of the solution of NPE, the Neumann problem is reduced to the solution of *allied* Dirichlet problem for NPE.