Harmonic maps from polyhedra to sphere with tangent boundary conditions on faces

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We consider maps from a convex polyhedra $P \subset R^3$ to a unit sphere $S^2 \subset R^3$ which are tangent to faces (i.e. faces of P are mapped to grand circles of S^2 , parallel to the face,) and which are continuous away from vertices of P. We obtain complete homotopy classification of such maps. We obtain lower bounds for harmonic map energy of such maps in terms of homotopy invariants. For the case of reflectionsymmetric maps on a rectangular prism, we obtain upper and lower energy bounds, whose ratio is bounded by a constant, depending from aspect ratios of the prism. We observe that for some homotopy types of maps on a rectangular prism, there is a transition from smooth to singular-on-edge minimizing energy configuration, as a function of aspect ratios of the prism. This is joint work with A. Majumdar and JM Robbins. It is motivated in part by applications to bi-stable nematic liquid crystal displays.