The Growth Rate of Random Fibonacci-Type Sequences

Hei-Chi Chan

University of Illinois at Springfield, USA

Estimating the growth rate of random Fibonacci-type sequences is both challenging and fascinating. In this talk, I will discuss a recent result in this area. Let a denote an infinite sequence of natural numbers $\{a_1, a_2, \cdots\}$ and define a random Fibonacci-type sequence by $F_{-1} = 0$, $F_0 = 1$, $a_0 = 0$, and

$$F_k = 2^{a_k} F_{k-1} + 2^{a_{k-1}} F_{k-2}$$

for $k \geq 1$. Then, for almost all such infinite sequences a, we have

$$\lim_{n \to \infty} \frac{\ln F_n}{n} = 1.30023 \cdots .$$

This result is motivated by the work of Embree, Trefethen, Viswanath and Wright. The main ingredient is the invariant measure of a dynamical system defined on the interval [0, 1).