Darboux transformations for the matrix Schrödinger equations

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The intertwining operator technique is applied to the matrix Schrödinger equation. The first- and second-order matrix Darboux transformations, factorization, supersymmetry, chains of transformations are studied. A relation between the matrix first-order Darboux transformation and supersymmetry is considered. The main differences between the matrix supersymmetry and the standard scalar supersymmetry for one Schrödinger equation and for the coupled systems of equations are discussed. An integral representation of the differential 1st and 2nd order transformation operators is given. It is shown that in a particular case the second-order integral transformations turn into expressions for matrix solutions and potentials obtained by the inverse scattering problem with degenerate integral kernels. The relationships between the different potential matrices and their pertinent solutions are obtained in a more general case when the second-order transformations are simultaneously realized on M states. Let us note, there is an appreciable difference between the 2nd order matrix and the scalar Darboux transformations. It is shown that there is a wide class of exactly solvable systems of Schrödinger equations obtained within the 2nd order transformation which cannot be constructed through the 1st order transformations. The 2nd order transformation can be realized with a transformation vector-function, while the 1st order transformation with a vector-function is impossible.