On an asymptotic boundary value problem for second order differential equations

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Asymptotic boundary value problems for second order ordinary differential equations appear when we look for radial solutions of nonlinear elliptic equations in unbounded domains with prescribed behaviour at infinity. Here, we are seeking a solution which has a bounded derivative at infinity and satisfies the Dirichlet or Neumann conditions on the boundary of the ball. It gives the following boundary value problems:

$$x'' = f(t, x, x'), \quad \text{for} \quad t \in (0, \infty), \quad x(0) = 0 = \lim_{t\to\infty} x'(t)$$

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The first of them is nonresonant – the problem $x'' = 0, \quad x(0) = 0 = \lim_{t\to\infty} x'(t)$ has no nontrivial solutions. The existence of at least one solution is proved by using topological methods.

The second problem is resonant, since the corresponding homogeneous linear problem: $x'' = 0, \quad x'(0) = 0 = \lim_{t\to\infty} x'(t)$ has nontrivial solutions – constant functions. The existence of at least one solution is proved by using Miranda's Theorem.