The von Neumann problem and singular perturbations of nonnegative operators

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We develop a new approach and present a solution to von Neumann's problem on parametrization in explicit form of all nonnegative self-adjoint extensions of a densely defined nonnegative symmetric operator. Our formulas are based on the Friedrichs extension and also provide a description for closed sesquilinear forms associated with nonnegative self-adjoint extensions. All basic results of the well-known Kreĭn and Birman-Vishik theory and its complementations are derived as consequences from our new formulas, including the parametrization in terms of classical von Neumann's formulas of all domains of nonnegative self-adjoint extensions as well as canonical resolvents. As an application all nonnegative quantum Hamiltonians corresponding to point-interactions in $\mathbb{R}^3$ are described. For a nonnegative self-adjoint operator A we establish for the first time necessary and sufficient conditions on singular perturbation G that results the existence of nonnegative self-adjoint operators among the realizations of $A+G$, as well as when those nonnegative realizations contain the Krein-von Neumann nonnegative extremal self-adjoint extension. Singular perturbations of the Laplace operator in $\mathbb{R}^3$ by delta potentials are considered. The talk is based on joint work with Yu.Arlinskii.