On Krasovskii Criterion of Optimal Stabilization

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Consider a system of differential equations of perturbed motion

$$\dot{x} = X(t, x; u) \tag{1}$$

where $x = (x_1, ..., x_n)$, $X = (X_1, ..., X_n)$, $u = (u_1, ..., u_r)$ -is a control function, and $X(t, x; u)$-is an almost periodic function in t.

Suppose that function $X(t, x; u)$ is defined, continuous and satisfying Lipschitz condition in x in the domain

$$t \in R, \|x\| < H \ (H = const). \tag{2}$$

Suppose that criterion of motion $x(t)$ is given as

$$I = \int_{t_0}^{\infty} \omega(t, x_1[t], ..., x_n[t]; u_1[t], ..., u_r[t]) \, dt, \tag{3}$$

where $\omega(t, x, u)$ is nonnegative function.

Theorem. If there exist almost periodic in t, positive definite, continuously differentiable function $V^0(t, x)$ and functions $u_j^0(t, x)$, which are satisfying in the domain (2) the following conditions:

1) function $w(t, x) = \omega(t, x; u^0(t, x))$ is nonnegative, and $w(t, x)$ may equal zero only in the points of set which does not include any samitrajectory of the system (1) $x(x_0, t_0, t), (t_0 < t < +\infty)$ entirely (except the trivial solution);

2) an equality

$$B[V^0; t, x; u^0(t, x)] = 0 \tag{4}$$

holds, where

$$B[V; t, x; u] = \frac{\partial V}{\partial t} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} X_i(t, x; u) + \omega(t, x; u) = \frac{dV}{dt} + \omega(t, x; u); \tag{5}$$

3) an equality

$$B[V^0; t, x; u] \geq 0, \tag{6}$$

holds for each control functions u_j, then functions $u_j^0(t, x)$ solve the problem of optimal stabilization and an equality

$$\int_{t_0}^{\infty} \omega(t, x^0[t]; u^0[t]) \, dt = \min \int_{t_0}^{\infty} \omega(t, x[t]; u[t]) \, dt = V^0(t_0, x(t_0)) \tag{7}$$

holds.