Cauchy Problem for Schroedinger Equation

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Let E^3 be a three-dimensional real Euclidean space, $y = (y_1, y_2, y_3)$, $x = (x_1, x_2, x_3) \in E^3$. D bounded single-connected domain in E^3 with boundary ∂D, consisting of a compact connected part T of the plane $y_3 = 0$ and smooth Lyapunov surface S, laying in the semispace $y_3 \geq 0$, $\partial D = S \bigcup T$, $\bar{D} = D \bigcup \partial D$; $\frac{d}{dn}$ - operator of differentiation on external normal, $\lambda(y) \in C^2(\bar{D})$. Δ is a three-dimensional Laplace operator

Problem (Cauchy). *Let $U(y) \in C^2(D)$ and*

$$\Delta U(y) - \lambda^2(y)U(y) = 0, \qquad y \in D \tag{1}$$

$$U(y) = f(y), \qquad \frac{\partial U}{\partial n}(y) = g(y), \quad y \in S \tag{2}$$

where $f(y)$ and $g(y)$ are given functions of the classes $C^1(S)$ and $C(S)$, respectively. It is required to compute $U(y)$ in interior points of D.

Since specified problem is ill-posed we will assume that solution exists, belongs to the class $C^2(D) \bigcap C^1(\bar{D})$ and satisfies the condition

$$|U(y)| + \left| \frac{d}{dn} U(y) \right| \leq M, \qquad y \in T,$$

where M given positive number.

In this assumption we shall build approximate solution for the problem (1)-(2).

Denote

$$-\frac{2}{\pi^2} e^{\sigma x_3^2} \Phi_\sigma(x, y) = \int\limits_0^\infty Im \left[\frac{e^{\sigma w^2}}{w - x_3} \right] \frac{\cos \lambda u du}{\sqrt{u^2 + s}},$$

$$G_\sigma(x,y) = \Phi_\sigma(x,y) + e^{-\sigma x_3^2} \int\limits_D \Gamma(x,t)\Phi_\sigma(t,y) e^{\sigma t_3^2} dt, \; U_{\sigma\delta}(x) = \int\limits_S \left[g_\delta G_\sigma - f_\delta \frac{dG_\sigma}{dn} \right] ds_y,$$

here $t = (t_1, t_2, t_3) \in R^3$ $s = (y_1 - x_1)^2 + (y_2 - x_3)^2$, $w = i\sqrt{u^2 + s} + y_3$, $\Gamma(x,t)$ is a resolvent of the kernel of the integral equation of special kind [1]. $f_\delta(y), g_\delta(y) \in C(S)$ and $\sup\limits_S |f(y) - f_\delta(y)| + \sup\limits_S |g(y) - g_\delta(y)| < \delta$, $\sigma = \frac{1}{x_0} ln \frac{M}{\delta}$, $x_0 = \max\limits_{\bar{D}} x_3$, $\delta < Mexp(-c|D|)$, $|D|$- volume of D and $c = const > 0$. Under this conditions the following inequality holds

$$|U(x) - U_{\sigma\delta}| \leq \Psi(\sigma) M^{1 - \frac{x_3}{x_0}} \delta^{\frac{x_3}{x_0}}, \qquad \Psi(\sigma) = O(\sqrt{\sigma}), \; \sigma \to \infty,$$

and

$$\lim_{\delta \to 0} U_{\sigma\delta} = U(x).$$