Asymptotic properties of the generalized eigenfunctions of relativistic Schrödinger operators

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This talk is devoted to a discussion about asymptotic properties of the generalized (or "continuum") eigenfuctions $\varphi^{\pm}(x,\,k)$ of relativistic Schrödinger operators $H = \sqrt{-\Delta} + V(x)$ in $\mathbf{R}^3$ with $V(x)$ decreasing rapidly at infinity. Based on the construction of $\varphi^{\pm}(x,\,k)$ via the limiting absorption principle for H, we show that: (i) $\varphi^{\pm}(x,\,k)$ are distorted plane waves; (ii) $\varphi^{\pm}(x,\,k)$ are characterized as solutions to the Lippmann-Schwinger type integral equations, of which kernels have severe singularity. We also show that $\varphi^{\pm}(x,\,k)$ are the sums of plane waves and solutions to the equations $(\sqrt{-\Delta}-|k|)u = f$ with radiation conditions, and discuss regularity of $\varphi^{\pm}(x,\,k)$ with respect to the variable x. Under a more restrictive condition on V, we show that $\varphi^{\pm}(x,\,k)$ are asymptotically the sums of plane waves and spherical waves.