Chaos and the coexistence of attractors in a model system of coupled difference equations

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The dynamical behavior of a semi-discrete model of conservation laws featuring interactions in chain is studied in two different parameter regions. The model corresponds to a partial differential equation of conservation laws. For sinusoidal forcing term it is represented by two coupled nonlinear difference equations of the first order, namely

$$x(k+1) = f_1(x(k), y(k), w) \tag{1}$$

$$y(k+1) = f_2(x(k), y(k), w) \tag{2}$$

where nonlinear functions $f_1(.)$ and $f_2(.)$ satisfy a set of appropriate conditions and w is taken as an external control parameter. The paper aim is focussed on the complexity in system behavior when the external control parameter is varied. The coexistence of attractors between various complex oscillations and between periodic oscillations and chaos has been found. The considered problem is characterized by the following classes of trajectories: i) quasi-periodic trajectories characterized by an irrational number of rotations W which are related to non-commensurable structures, appearing in result of external excitation with period T_0 on the lattice with non-commensurable period W. When a critical values T_c is attained then the continuity of the trajectories is abrupt; ii) periodic trajectories with $n \geq 3$ for which the conditions $x(k+n) = x(k)$ and $y(k+n) = y(k)$ are fulfilled; iii) homo- and hetero- cyclic trajectories describing structures, which asymptotically go to periodic at $k \rightarrow \infty$ but containing for limited k some deformed distributions; iv) choatic trajectories with corresponding structures having some disorder. The chaos in such models as that studied in this paper appears very specific and estimated changes in the variable distributions as a rule are due to the tripling and/or fourthing of the elementary structures.