Exponentially Small Error Estimates of Quasiclassical Eigenvalues

Julio Toloza
Virginia Tech

We study the behavior of truncated Rayleigh-Schrödinger series for the low-lying eigenvalues of the time-independent Schrödinger equation, in the semiclassical limit $\hbar \searrow 0$. We prove that, under certain hypotheses on the potential $V(x)$ and for small $\hbar > 0$, the perturbation series for the eigenvalues admits an exponentially accurate truncation. That is, the difference between the truncated series and the actual eigenvalue can be made smaller than $\exp(-C/\hbar)$ for a positive constant C. An analogous statement is shown for the corresponding eigenfuctions.