Discontinuous Solutions of Hamilton-Jacobi Equations: Existence, Uniqueness and Regularity.

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In this talk, we address the discontinuous solutions of Hamilton-Jacobi equations. The existence of L^∞ solutions is proven for general Hamiltonians. Then we clarify the connections in between the existing notions including the classical semicontinuous viscosity solution by Ishii. We look at the important special class of Hamiltonians and show the uniqueness of discontinuous solutions including the classical semicontinuous viscosity solutions, L^∞ solutions. We prove the new interesting regularity property for locally strictly convex Hamiltonians H such as $H(p) = (1+|p|^2)^{\frac{1}{2}}$. In other words, with discontinuous initial data, the discontinuous solutions of $u_t + H(Du) = 0$ becomes Lipchitz continuous in finite time. The difference between the regularization effect of $u_t + |Du|^2 = 0$ and $u_t + (1+|Du|^2)^{\frac{1}{2}} = 0$ is similar to the difference between the difference of regularity of $u_t - \Delta u = 0$ and $u_t - \Delta u^2 = 0$.