Schrödinger operator on homogeneous metric trees: spectrum in gaps

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We study the spectral properties of the Schrödinger operator $A_{gV} = A_0 + gV$ on a homogeneous rooted metric tree, with a decaying real-valued potential V and a coupling constant $g \geq 0$. The spectrum of the free Laplacian $A_0 = -\Delta$ has a band-gap structure with a single eigenvalue of infinite multiplicity in the middle of each finite gap. The perturbation gV gives rise to extra eigenvalues in the gaps. These eigenvalues are monotone functions of g if the potential V has a fixed sign. Assuming that the latter condition is satisfied and that V is symmetric, i.e. depends on the distance to the root of the tree, we carry out a detailed asymptotic analysis of the counting function of the discrete eigenvalues in the limit $g \to \infty$. Depending on the sign and decay of V, this asymptotics is either of the Weyl type or is completely determined by the behaviour of V at infinity.

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