Regularization of nonlinear ill-posed problems by dynamical system method

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Let F be a nonlinear twice Fréchet differentiable map in a Hilbert space H and the equation

$$F(z) = 0 \tag{1}$$

be solvable (maybe nonuniquely). Assume that y is a solution to (1), and $F'(y)$ is not boundedly invertible. In this case solving (1) is an ill-posed nonlinear problem. A new approach to solving such a problem is presented. The approach is based on a construction of a dynamical system

$$\dot{u} = \phi(u, t), \quad u(0) = u_0, \tag{2}$$

with a unique global solution $u(t)$ for any $u_0 \in B(y, r)$ that tends to an element $u(\infty)$ (in the norm of H), and $F(u(\infty)) = 0$.

If y is a unique solution to (1) in the ball $B(y, r)$, then $u(\infty) = y$. Otherwise $u(\infty)$ depends on the choice of u_0.

The approach is justified, that is, the existence and uniqueness of the global solution to (2) with a suitable choice of ϕ and u_0 is proved.