Asymptotics of bound states, bands and resonances for laterally coupled quantum waveguides and layers

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Spectral problem for the Dirichlet Laplacian in two-dimensional strips (three-dimensional layers) of widths d_+, d_-, $d_+ > d_-$ coupled through small openings $a\omega_i, i = 1, ...n$, (a is small parameter) is considered. Method of matching of asymptotic expansions of solutions of boundary value problem is used. The asymptotics (in a) of a bound state λ_a close to the threshold π^2/d_+^2 is obtained:

$$\lambda_a = \{ \begin{array}{ll} \frac{\pi^2}{d_+^2} - (\frac{\pi^3}{d_+^3}\sum_{i=1}^n c_{\omega_i})^2 a^4 + o(a^4), & \mathbf{R}^2, \\ \frac{\pi^2}{d_+^2} - (\frac{1}{d_+^2}\exp(-\frac{2d_+^3}{3\pi^2}(\sum_{i=1}^n b_{\omega_i})^{-1}a^{-3}(1+o(1))), & \mathbf{R}^3. \end{array}$$

Here c_{ω_i} is a capacity of ω_i in $\mathbf{R}^2$, b_{ω_i} is an average virtual mass of ω_i in $\mathbf{R}^3$. The case of two identical waveguides (layers) is considered too.

The asymptotics of resonances close to $N-$th threshold is determined for 2D case.. 2D waveguides in a magnetic field and curved waveguides are considered.

Asymptotics of bands for the case of periodic system of coupling windows (period L) for two waveguides is constructed:

$$[\frac{\pi^2}{d_+^2} - \frac{3\pi^3}{2Ld_+^3}a^2 + o(a^2), \frac{\pi^2}{d_+^2} - \frac{\pi^3}{2Ld_+^3}a^2 + o(a^2)], \quad \mathbf{R}^2.$$

$$[\lambda_1^+ - \frac{18\pi(\psi_1^{+0})^2 b_\omega}{L}a^3 + o(a^3), \lambda_1^+ - \frac{6\pi(\psi_1^{+0})^2 b_\omega}{L}a^3 + o(a^3)], \quad ,\mathbf{R}^3.$$

Here λ_1^+ is the first transversal eigenvalue, ψ_1^{+0} is the value of the normal derivative of the corresponding transversal eigenfunction at the centre of the opening. There is a gap for sufficiently small a. For the case of layers coupled through singly (Λ_1) and doubly (Λ_2) periodic system of windows there is no gap, and the asymptotics of the lower bound for the continuous spectrum is:

$$\lambda_{min} = \frac{\pi^2}{d_+^2} - \frac{\pi^6 b_\omega^2}{Ld_+^6}a^6 + o(a^6), \quad \text{for} \quad \Lambda_1,$$

$$\lambda_{min} = \frac{\pi^2}{d_+^2} - \frac{\pi b_\omega|\hat{\Lambda}|}{d_+^3}a^3 + o(a^3), \quad \text{for} \quad \Lambda_2,$$

where $|\hat{\Lambda}|$ is a square of the Brillouin zone for Λ_2.