Stability of a Model of Relativistic Quantum Electrodynamics

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The relativistic "no pair" model of quantum electrodynamics uses the Dirac operator, $D(A)$ for the electron dynamics together with the usual self-energy of the quantized ultraviolet cutoff electromagnetic field A — in the Coulomb gauge. There are no positrons because the electron wave functions are constrained to lie in the positive spectral subspace of some Dirac operator, D, but the model is defined for any number, N, of electrons, and hence describes a true many-body system. In addition to the electrons there are a number, K, of fixed nuclei with charges $\leq Z$. If the fields are not quantized but are classical, it was shown earlier (with Siedentop and Solovej) that such a model is always unstable (the ground state energy $E = -\infty$) if one uses the customary $D(0)$ to define the electron space, but is stable ($E > -\text{const.}(N + K)$) if one uses $D(A)$ itself (provided the fine structure constant α and Z are not too large). This result is extended to quantized ultraviolet cutoff electromagnetic fields here, and stability is proved for $\alpha = 1/137$ and $Z \leq 42$. This formulation of QED (using the spectrum of $D(A)$ instead of $D(0)$ to define an electron) is somewhat unusual because it means that the electron Hilbert space is inextricably linked to the photon Fock space. But such a linkage appears to better describe the real world of photons and electrons. (joint work with Michael Loss)