The Efimov effect of three-particle Schrödinger operators on a lattice. Asymptotics for the number of eigenvalues

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The Efimov effect is one of the most remarkable results in the spectral analysis for three-particle Schrödinger operators on Euclidean space. Roughly speaking,the effect will be explained as follows:

If none of the three two-particle subsystem has bound states with negative energy, but at least two of them have a resonance state with zero energy, then the three-particle system has an infinite number of three-particle bound states with negative energy, accumulating at zero.

This effect first discovered by Efimov in [1]. The mathematically rigorous proof of the result has been presented(see for instance [2-3]).

We consider a system of three-particle,which move in three-dimensional lattice Z^3 and interact through a pair short-range negative potentials. The Hamiltonian of this system has form

$$H = \triangle_i - \sum_{i<j} V_{ij},$$

where $\triangle_i$ is the lattice Laplacian and V_{ij} in the coordinate representation is the multiplication operator by function $V_{ij}(r_i - r_j), V_{ij} \geq 0$. We reduce the investigation of the spectral properties of the operator H to the analysis of the family of self-adjoint,bounded operators (the three-particle discrete Schrödinger operators) $\{H(K), K \in T^3\}$, acting in the Hilbert space of square-integrable functions defined on the torus, applying the Fourier transform and the decomposition into the direct operator integrals.

We assume that for the two-particle discrete Schrödinger operators $h_\alpha(k), k \in T^3, \alpha = 1, 2, 3$ some natural assumptions are fulfilled and establish the following results:

Let $N_0(z)$ be the number of eigenvalues of the three-particle Schrödinger operator $H(0)$ lying on the left from point $z < 0$.

(i)The $N_0(z)$ obeys asymptotics

$$\lim_{z \to 0} \frac{N_0(z)}{|\log |z||} = C_0 > 0,$$

where the $C_0 > 0$ depend only on the masses of the particles.

Let $U_0 \subset T^3$ be sufficiently small punctured neighborhood of the point $0 \in T^3$.

(ii) For any $K \in U_0$ the operator $H(K)$ has only finite number eigenvalues lying on the left-hand side of the essential spectrum.

We also shall point to some important spectral properties of the operators $H(K), K \in T^3$,which are analogously or different for both of the Schrödinger operators on Euclidean space and lattice.

[1]V. Efimov: Nucl.Phys. A.210 (1973),157-158. [2]D. R. Yafaev: Math.USSR-Sb. 23 (1974),535-559. [3] A. V. Sobolev: Comm.Math. Phys. 156 (1993), 127–168.