An approach to solving boundary-value problems of elasticity in coordinate systems with incomplete separation of variables

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We present an approach to constructing exact analytical solutions to the displacement boundary-problem of elasticity for bodies that are described by surfaces in cyclidal coordinate systems with incomplete separation of variables.

We consider the displacement boundary-value problem of elasticity formulated in a simple sub-family of cyclidal coordinates that includes bi-spherical and toroidal coordinate systems.

One of the key points in our approach is the developed form of general solution for the elastostatics (Lamé) equation, where a solution is represented as a linear combination of a scalar and vector harmonic functions. These functions have to satisfy not only boundary conditions, but also additional differential constraint. It is shown that this general solution reduces the boundary-value problem to functional or algebraic equations of the simplest form.

As an example, exact analytical solution of the displacement boundary-value problem for a toroidal elastic body is presented. According to the approach, the original boundary-value problem is reduced to a series of infinite linear algebraic equations with three-diagonal matrices. An analytical technique for solving systems of diagonal form is developed.

General aspects of uniqueness of solution of generalized Cauchy-Riemann equations in double-connected spatial toroidal regions are considered, and the uniqueness of the obtained solution is proved.