Method of Supplementary Argument

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Implementation of the method of supplementary argument for the equations of the first order was considered in [1]. In [2] it was shown the implementing of this method for the equation of third order. In this article the method of supplementary argument is implemented for the systems of quasilinear equations. By the method of supplementary argument the problem is reduced to integral equation, in which auxiliary functions are defined from the system of equations without right parts with matched initial conditions. The systems of equations of

(1)
$$u_t(t,x) + g(t,x,u(t,x))u_x(t,x) = f(t,x,u(t,x)),$$

type with the initial condition

(2)
$$u(0,x) = \varphi(x), \quad (t,x) \in R_+ \times R$$

are considered.

Theorem. Let vector-function $\varphi(x) \in \overline{C}(\mathbb{R}^n) \cap Lip(L_1)$,

$$f(t, x, u) \in C^{(0,1,1)}(R_+ \times R \times R) \cap Lip(L(t)|_x, L(t)|_u),$$

$$g(t, x, u) \in C^{(0,1,1)}(R_+ \times R \times R) \cap Lip(L(t)|_x, L(t)|_u),$$

$$\int_0^\infty L(s)ds(2+L_1) \le \alpha < 1.$$

Then the problem (1) - (2) has the unique solution $u(t,x) \in C^{(1,1)}(R_+ \times R)$, which is presented as

(3)
$$u(t,x) = \varphi(x) - \int_{0}^{t} g(t, p(s,t,x), v(s,t,x)) ds + \int_{0}^{t} f(s, p(s,t,x), v(s,t,x)) ds,$$

where the vector-function $\nu(\tau,t,x)$ and scalar function $p(\tau,t,x)$ is determined from the system

$$v_t(\tau, t, x) + g(t, x, v(t, t, x))v_x(\tau, t, x) = 0, \quad v(t, t, x) = u(t, x),$$

 $p_t(\tau, t, x) + g(t, x, v(t, t, x))p_x(\tau, t, x) = 0, \quad p(t, t, x) = x,$

and it is the solution of the problem (1)-(2) with the same properties.

References

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