On the singular Cauchy type integrals with a continuous density for function of many variables

ABDURASUL GAZIEV
Department of Mathematics Samarkand State University , University Boulevard 15, 703004 , Samarkand , Uzbekistan

The singular integral operators are met in the solution of many important problems of applied character; particularly, in solution of boundary problems of analytical functions, linear and non-linear singular integral equations, various problems of the mathematical physics, theory of elasticity, queuing problems, etc.

At the present time the properties of the singular Cauchy type integral in various functional spaces for one-dimensional case are rather well studied. At the same time the properties of Cauchy type integral for multi-dimensional case (on a continuity up io the boundary of a semi- cylindrical domain; on a behavior, at the boundary of the semi-cylindrical domain, of limiting values of Cauchy type integral that are expressed by Sokhotskii type formulas via repeated singular integral) are not well investigated.

In the present paper we consider a multiple Cauchy type integral

$$F(z) = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{f(\tau)}{\tau - z} d\tau, \tag{1}$$

where $z = (z_1, z_2, ..., z_n)$, $\tau = (\tau_1, \tau_2, ..., \tau_n)$, $d\tau = d\tau_1 d\tau_2 ..., d\tau_n$,

$$\tau - z = \prod_{k=1}^{n} (\tau_k - z_k), \quad \Delta = \gamma^1 \times \gamma^2 \times \cdots \times \gamma^k$$

γ^k is a closed Jordon rectifiaable curve (c. j. r. c) on the complex plane z_k $(k = \overline{1,n})$, $f \in C_\Delta$, where C_Δ is a space of continuous functions on the spanning set Δ.

To study the properties of the limiting values of integral (1) the partial and mixed moduli of continuity of function f, and a new characteristic

$$\theta(\delta) \quad (\theta^k(t_k, \delta) = \int_{\gamma^k_\delta(t_k)} |d\tau_k|, \quad \theta^k_{(\delta)} = \sup_{t_k \in \gamma^k} \theta^k(t_k, \delta), \gamma^k_\delta(t_k) = \{\tau \in \gamma^k;$$

$$|\tau - t_k| \leq \delta, \quad \delta \in (0, d_k), \quad d_k = \sup(|\tau_k - t_k|))$$

of the curve γ^k were chosen.

With the help of these characteristics we obtained inequalities analogous to those of Zygmund type, which connect the partial and mixed moduli of the continuity of an image and pre-image of the limiting value of integral (1) for any (c. j. r. c) γ.

With the help of the obtained inequalities we constructed the Banach space, which is invariant with respect to the limiting value of integral (1), which is spread to a class of curves, satisfying the condition $\theta^k(\delta) \sim \delta$ $(k = \overline{1,n})$. This class is significantly wider than that of piecewise smooth curve (the presence of cusps is supposed).