Hopf's last hope: Spatiotemporal chaos in terms of unstable recurrent patterns

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If everthing in a turbulent systems is in constant flux, how is it that humans are able to distinguish different kinds of turbulence? Hopf's answer was that dynamics drives a given spatially extended system through a repertoire of unstable patterns; as we watch a given "turbulent" system evolve, every so often we catch a glimpse of a familiar pattern. For any finite spatial resolution, the system follows approximately for a finite time a pattern belonging to a finite alphabet of admissible patterns, and the long term dynamics can be thought of as a walk through the space of such patterns, just as chaotic dynamics with a low dimensional attractor can be thought of as a succession of nearly periodic (but unstable) motions.

Hopf's proposal is in its spirit very different from most ideas that animate current turbulence research. It is not the Kolmogorov's 1941 homogeneous turbulence with no coherent structures fixing the length scale, here all the action is in specific coherent structures. It is emphatically not universal; spatiotemporally periodic solutions are specific to the particular set of equations and boundary conditions. And it is not probabilistic; everything is fixed by the deterministic dynamics with no probabilistic assumptions on the velocity distributions or external stochastic forcing.

I will describe a modest implementation of Hopf's program on a 1-dimensional spatially extended Kuramoto-Sivashinsky system, a PDE that describes interfacial instabilities such as unstable flame fronts.