Multidimensional Transonic Shocks and Free Boundary Problems for the Euler Equations for Potential Fluids

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We will first discuss recent results on the existence and stability of multidimensional transonic shocks (hyperbolic-elliptic shocks) for the Euler equations for steady potential fluids in infinite cylinders. The Euler equations, consisting of the conservation law of mass and the Bernoulli law for velocity, can be written as a second-order, nonlinear elliptic-hyperbolic equation of mixed type for the velocity potential. The transonic shock problem in an infinite cylinder can be formulated into the following free boundary problem: The free boundary is the location of the multidimensional transonic shock which divides two regions of smooth flow in the infinite cylinder, and the equation is hyperbolic in the upstream region where the smooth perturbed flow is supersonic. We will introduce a nonlinear approach to deal with such a free boundary problem in order to solve the transonic shock problem in unbounded domains. Our results indicate that there exists a unique solution of the free boundary problem such that the equation is always elliptic in the unbounded downstream region, the velocity state at infinity in the downstream direction is uniquely determined by the given hyperbolic phase, and the free boundary is smooth, provided that the hyperbolic phase is close to a uniform flow; and the free boundary is also stable under the steady perturbation of the hyperbolic phase. We will also present some further results in this direction, including the results on the existence and stability of multidimensional transonic shocks near spherical or circular transonic shocks in unbounded domains (joint work with Mikhail Feldman).