On the Cauchy problem for a dynamical Euler's elastica

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We consider the dynamics for a thin, closed loop inextensible Euler's elastica moving in three dimensions. The equations of motion for the elastica include a wave equation involving fourth order spatial derivatives, and a second order elliptic equation for its tension. Local existence and uniqueness of solutions are established for initial data in suitable Sobolev spaces.

The proof uses a Hasimoto transformation to rewrite the equations in convenient coordinates for the space and time derivatives of the tangent vector. A feature of this elastica is that it exhibits time-dependent monodromy – a frame parallel-transported along the elastica is in general quasi-periodic, resulting in time-dependent boundary conditions for the coordinates. We address this complication by a gauge transformation, after which a contraction mapping argument can be applied.

(Joint work with L. E. Thomas.)