Interpolation and Inverse Spectral Theory

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We are interested in finding necessary and sufficient conditions for interpolation at irregular points to hold. Given a sequence of real numbers $\{\mu_n\}_{n\geq 0}$, can we find a space containing a sequence $\{S_n(\mu)\}_{n\geq 0}$ such that the values $\{F(\mu_n)\}_{n\geq 0}$ define a unique function by the following sampling expansion formula, $F(\mu) = \sum_{n\geq 0} F(\mu_n)S_n(\mu)$? We show that the inverse spectral problem, and more precisey the Gelfand Levitan theory, can be used to construct sampling type theorems from the knowledge of the sampling points only. Thus one can recover a function from the knowedge of its given values on a sequence of points. This improves Kramer's theorem as it reveals all possible distributions of the sampling points together with a construction of the sampling functions.