The Spectrum of Differential Operators with Almost Constant Coefficients

HORST BEHNCKE
University of Osnabruck

The absolutely continuous spectrum of differential operators of order $2n$ of the form

$$Ly = w^{-1}\sum_{k=0}^{n}(-1)^n(p_k y^{(k)})^{(k)}$$

on $\mathcal{L}^2((0,\infty),w)$ is determined. We assume w, $p_n > 0$ and that the real valued coefficients admit a decomposition

$$p_k = p_{k1} + p_{k2} + p_{k3} + p_{k4} \qquad k = 0,\ldots,n$$
$$w = w_1 + w_2 \quad p_{n3} = p_{n4} = 0$$

with

$$[p_{k3}, q_k, p'_{k,2}, p''_{k1}\gamma^{-1}, p'^{2}_{k,1}w^{-1}\gamma^{2k-1}], \gamma^{2k}w^{-1}w'_2 w^{-1}, w''_1 w^{-1}, w'^{2}_1 w^{-1}\gamma^{-1} \in \mathcal{L}_1$$
$$q_k\gamma^{2k}w^{-1}, p'_{k,1}\gamma^{2k-1}w^{-1}, w'_1 w^{-1}\gamma^{-1} \to 0 \qquad x \to \infty$$

where $q_k = \int_\infty^t p_{k4}$ and $\gamma = (wp_n^{-1})^{1/2n}$. Moreover assume $(p_{k,1}+p_{k,2})\gamma^{2k}w^{-1} \to c_k$.

Then the absolutely continuous part of the spectrum of any selfadjoint extension of L is unitarily equivalent to that of the operator multiplication with $\sum_{k=0}^{n} c_k x^{2k}$ on $\mathcal{L}^2(0,\infty)$.

This theorem extends all known results for such operators. The proof is based on a variant of the Liouville-Kummer transformation and asymptotic integration.

Various extensions of this result are also considered.