Asymptotic expression of negative eigenvalue number of an operator differential equation with even order according to small parameter having weighted function in eigenvalue

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Let H be seperable Hilbert space and while $\rho(x) > 0$ is a continuous function in $[0, \infty)$. Let H_1 a Hilbert space of H-valued measurable function $f(x)$, $(0 \leq x < \infty)$ that satisfies $\int_0^\infty \rho(x) \|f(x)\|^2 dx < \infty$ condition.

Let L_h, an operator generated by differential expression

$$\ell_h(y) = \frac{1}{\rho(x)} \{(-1)^n h y^{(2n)} - Q(x) y\}, \qquad (0 \leq x < \infty)$$

and boundary conditions

$$y^{(j)}(0) = 0, \qquad j = 0, 1, ..., n-1$$

in H_1 space.

$Q(x)$ in $\ell_h(y)$ is an operator function which satisfied $Q^*(x) = Q(x) \geq 0, \quad Q(x) \in \sigma_\infty$, $\lim_{x \to \infty} \|Q(x)\| = 0$ and $h > 0$ is small parameter. Under this conditions, negative spectrum of L_h operator consists of eigenvalues.

Let $N_h(\epsilon)$ denoted numbers of negative eigenvalues of L_h that smaller than $-\epsilon$, $(\epsilon > 0)$.

When $h \to 0$

$$N_\epsilon(h) \sim \frac{1}{\pi \sqrt[2n]{h}} \sum_j \int\limits_{\alpha_j(x) > \epsilon} \rho(x) \sqrt[2n]{\alpha_j(x) - \epsilon} dx$$

is obtained. Here, $\alpha_1(x) \geq \alpha_2(x) \geq ...$ are eigenvalues of $Q(x)$. While $n = 1$, asymptotic expression of $N_h(\epsilon)$ is found in [1].

REFERENCE

1. M.Bayramoğlu,I.Albayrak, Second International Symposium on Mathematical and Computational Applications, Baku, September 1-3,1999.

Joint work with Oya Baykal and Mehmet Bayramoğlu.