On the Cauchy problem for first-order elliptic systems

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In this note we present a formula for recovering the solution of systems of first-order differential equations of elliptic type of a special class in a domain on the basis of their values on the boundary.

Let $A_{l\times n}(x)$ denote the class of matrices $D(x^T)(x^T = (x_1, ..., x_m)^T$ the transposed column vector of x) with constant coefficients in C^1 for which

$$D^*(x^T)D(x') = \epsilon((\mid x \mid^2 + \lambda)U^0,$$

where $D^*(x^T)$ is the Hermitian conjugate of the matrix $D^*(x^T)$ and $\epsilon(x)$ is a diagonal matrix , U^0 is the unit vector.

We denote by G_ρ a domain in R^m whose boundary consists of part of the - surface of the cone

$$y_1^2 + ... + y_{m-1}^2 = \tau y_m^2, (\tau = \tan \pi/2\rho)$$

and a smooth surface S lying inside the cone.

We consider the system of differential equations (1)

$$D(\frac{\partial}{\partial x})u(x) = 0, \tag{1}$$

where the characteristic matrix

$$D(x^T) \in A_{l\times n}(x).$$

In this note we consider the Cauchy problem in the formulation of M.M. Lavrent'ev [1] . Suppose $u(x) \in C^1(G_\rho) \cap C(G_\rho)$ and u(x) satisfies (1) , and let $u_\delta(x)$ be continuous approximation to u(x) on S , i . e . ,

$$max \mid u(x) - u_\delta(x) \mid < \delta, \quad 0 < \delta < 1.$$

We want recover u (x) in G_ρ.

Theorem . Suppose u (x) is a solution of (1) and

$$\mid u(y) \mid = M, \quad y \in T = \partial G_\rho \setminus S,$$

where M is a given positive number . Set

$$u_\sigma(x) = \int_S N_\sigma(x, y)u(y)ds_y, \quad x \in \mathrm{G}_\rho.$$

Then

$$\mid u(x) - u_\sigma(x) \mid \leq MC(x)C(\sigma)exp(-\sigma\gamma^\rho), \quad x \in \mathrm{G}_\rho,$$

where

$$C(x) = C_\rho \int_S \frac{ds}{r^{m-1}},$$

$$C(\sigma) = \{ \begin{array}{ll} \sigma^m & for \quad m = 2k + 1, \quad k \geq 1, \\ \sigma^{m-1} & for \quad m = 2k, \quad k \geq 2. \end{array}$$

Corollari . The limit equality

$$\lim_{\sigma\to\infty} u(x) = u(x), \quad x \in G_\rho,$$

is satisfied uniformly in each compact set in G_ρ.

Bibliography

1 . M . M . Lavrent' ev , On some ill - posed problems of mathematical physics , Izdat . Sibirsk . Otdel Acad . Nauk . SSSR , Novosibirsk , 1962; English transl ., Springer - Verlag , Berlin , 1967 .