Euclidean Geometry Rediscovered

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University of Alabama at Birmingham

Greater Birmingham Mathematics Partnership
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The opinions expressed herein are those of the authors, and not necessarily those of the National Science Foundation.
The Book

Euclidean Geometry
— A Guided Inquiry Approach —

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SUNY Distinguished Professor of Mathematics, New Paltz, NY

Educational Advancement Foundation
2008, 2011
The Audience

- Mathematics majors
- No previous proofs-based course
- Pre-service high school math teachers
- Pre-service middle school math teachers
- Parallel graduate course for M.Ed. students.
The Dilemma

- Axiomatic Development
- Euclid
- Hilbert
- Pedagogy
Figure and Ground

“The individual perceives the environment as a total unit; he/she responds to the whole of what is seen and this whole is composed of the stimuli of which the person is aware (the figure) and those of which the person is not aware or does not attend (the ground).”

[Gestalt Center of Gainesville]
Figure and Ground

Ground
- Naïve logic and set theory
- Non-triviality
- Betweenness (for points and lines)
- Intersections (straightedge and compass)

Figure
- Congruence
- Definitions
- Constructions with straightedge and compass
- Eight axioms of plane geometry
- Theorems and proofs
Guided Inquiry
(Inquiry-Based Learning — IBL)

- The main work of the class meeting is problem-solving.
- Class goals emphasize development of skills such as communication and mathematical habits of mind.
- Most of the class time is spent on student-centered instructional activities, such as collaborative group work, student presentations, and discussion.
Guided Inquiry (IBL)

- The instructor’s main role is not lecturing, but guiding, asking questions, and giving feedback; student voices predominate in the classroom.
- Students and instructor share responsibility for learning, respectful listening, and constructive critique.
Course Daily Structure

Early in course
- Brief instructor introduction of topic
- Collaborative group work
- Processing as a whole class
- Homework
- Processing

Later in course
- “May be” brief introduction of topic
- Homework
- Small group discussion of homework
- Processing as a whole class
## Assessment

<table>
<thead>
<tr>
<th>Grade Element</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Quizzes (weekly)</td>
<td>40%</td>
</tr>
<tr>
<td>Participation (daily)</td>
<td>20%</td>
</tr>
<tr>
<td>Notebook (6 x / semester)</td>
<td>20%</td>
</tr>
<tr>
<td>Final Exam</td>
<td>20%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
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</tbody>
</table>
Notebook Grading

- Completeness: 6 points
- Selected Problem: 2 points
- Homework Attempt: 2 points

<table>
<thead>
<tr>
<th>Grading Cycle (6)</th>
<th>Median Grade</th>
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<tbody>
<tr>
<td>N1</td>
<td>8</td>
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<tr>
<td>N2</td>
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<tr>
<td>N3</td>
<td>8</td>
</tr>
<tr>
<td>N4</td>
<td>9</td>
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</tbody>
</table>

Kennesaw Mountain Undergraduate Mathematics Conference, Nov. 11, 2011
Definition. Figure $X$ is congruent to figure $Y$ if we can orient one, or a copy of one, on top of the other so that they match exactly.
Axioms

Axiom 1: Length Measure. Each segment $AB$ can be assigned a positive number $L(AB)$ called the length of $AB$, so that

i. The length of the unit segment is 1.

ii. Two segments are congruent if, and only if, they have the same length.

iii. If $A$, $B$, $C$ are three points with $B$ between $A$ and $C$, then $L(AB) + L(BC) = L(AC)$. 
Axioms 2 and 3 and Justified Constructions

- Axiom 2: SSS
- Axiom 3: SAS
- Problem 22: Construct the bisector of a given angle.
- Problem 25: Construct the midpoint of a segment.
From Constructions to Theorems and Proofs

Figure $\Leftrightarrow$ Ground

- Theorem 31. Every angle has a bisector.
- Theorem 34. Every segment has a midpoint.
- Theorem 35. The base angles of an isosceles triangle are equal.
- Theorems 43 & 44: ASA and AAS.
- Theorem 45. If two angles of a triangle are congruent, then the opposite sides are congruent.
Quiz 5

- **Theorem 35A.** The line segment connecting the vertex angle of an isosceles triangle to the midpoint of the base bisects the vertex angle and is perpendicular to the base.

- **Prove Theorem 35A.**

  Quizzes are open Book, but not open Notes.
# Quiz Rubric

<table>
<thead>
<tr>
<th>Explanation:</th>
<th>Accuracy:</th>
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<tbody>
<tr>
<td><strong>Using pictures, symbols, and/or vocabulary to convey the path to the identified solution</strong></td>
<td>Providing a complete and accurate solution appropriate for the given problem</td>
</tr>
<tr>
<td>3 Explanation is clear and complete</td>
<td>Solution is correct and complete with no errors</td>
</tr>
<tr>
<td>2 Explanation is clear, but not quite complete.</td>
<td>Solution is appropriate and demonstrates understanding, but is either not quite complete or contains a minor error</td>
</tr>
<tr>
<td>1 The explanation is partially complete and/or partially developed with gaps that have to be inferred</td>
<td>Solution is appropriate and demonstrates some understanding, but is either not complete or contains several minor errors</td>
</tr>
<tr>
<td>0 Does not achieve minimal requirements for 1 point</td>
<td>Does not achieve minimal requirements for 1 point</td>
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</tbody>
</table>

Adapted from the Oregon Department of Education’s 1995-2003 statewide assessments
## Quiz Rubric – 10 points Total

<table>
<thead>
<tr>
<th>Conceptual Understanding:</th>
<th>Evidence Of Problem Solving:</th>
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<tbody>
<tr>
<td><strong>Interpreting the concepts of the task and translating them into mathematics</strong></td>
<td><strong>Choosing strategies that can work, and then carrying out the strategies chosen.</strong></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>The translation of the task into adequate mathematical concepts using relevant information is completed</td>
</tr>
<tr>
<td>1</td>
<td>The translation of the major concepts of the task is partially completed and/or partially displayed</td>
</tr>
<tr>
<td>0</td>
<td>Does not achieve minimal requirements for 1 point</td>
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</table>
From Weak to Strong

Definition. Two lines are parallel if no point is on both lines.

Axiom 4 (Angle Non-Congruence). If point $B$ is in the interior of $\angle AXC$, then $\angle AXC$ is not congruent to $\angle AXB$.

Weak Alternate Interior Angle Theorem 39. If two lines have a transversal which forms alternate interior angles that are congruent, then the two lines are parallel.

– Figure ←→ Ground
A Hard Theorem?

- Definition. A tangent to a circle is a line that contains exactly one point of the circle.

- Theorem 52. Let $l$ be a line that contains a point $T$ of circle $c$ with center $O$. Then $l$ is tangent to $c$ if, and only if, the radius $OT$ is perpendicular to $l$. 
Quiz 10

Problem. Let $c$ be a circle with center $O$. Let $l$ be a line meeting $c$ at points $S$ and $T$ with $S$ not the same point as $T$. Draw radii $OS$ and $OT$. Show that neither $\angle OST$ nor $\angleOTS$ can be a right angle.

Use only theorems prior to Theorem 52.
Problem. Let $c$ be a circle with center $O$. Let $l$ be a line meeting $c$ at points $S$ and $T$ with $S$ not the same point as $T$. Draw radii $OS$ and $OT$. Show that neither $\angle OST$ nor $\angleOTS$ can be a right angle.

Use only theorems prior to Theorem 52.
Theorem 46. A triangle has at most one right angle.

Quiz 10

Problem. Let $c$ be a circle with center $O$. Let $l$ be a line meeting $c$ at points $S$ and $T$ with $S$ not the same point as $T$. Draw radii $OS$ and $OT$. Show that neither $\angle OST$ nor $\angle OTS$ can be a right angle.

Use only theorems prior to Theorem 52.
From Weak to Strong

- Axiom 5 (Parallel Lines). For every line $l$ and every point $P$ not on $l$, there is at most one line containing $P$ that is parallel to $l$.

- Strong Alternate Interior Angle Theorem 58. Assume a transversal intersects two lines. Then the two lines are parallel if, and only if, the alternate interior angles are equal.
Quiz Scores So Far

- Quizzes are scored on a 10 point scale (by a rubric).
- 75% is lowest “B” grade for course.

<table>
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<th>Quiz</th>
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<td>Quiz 10</td>
<td>10</td>
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Further Contact Information

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- David M. Clark  clarkd@newpaltz.edu
- Educational Advancement Foundation  http://eduadvance.org/
- The Legacy of R.L. Moore  http://legacyrlmoore.org/